Ergodicity-Breaking Reveals Time Optimal Economic Behavior in Humans

David Meder (davidm@drcmr.dk)

Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre, Kettegaard Allé 30
2650 Hvidovre, Denmark

Finn-Lennart Rabe (finn-lennart.rabe@hest.ethz.ch)

NCM Lab, ETH Zurich, Auguste-Piccard-Hof 1
8093 Zurich, Switzerland

Tobias Morville (tobias.morville@gmail.com)

Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre, Kettegaard Allé 30
2650 Hvidovre, Denmark

Kristoffer H. Madsen (stoffer@drcmr.dk)

Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre, Kettegaard Allé 30
2650 Hvidovre, Denmark

Magnus T. Koudahl (m.t.koudahl@tue.nl)

BIASlab, Eindhoven Univ. of Technology, Groene Loper 19
5612 AP Eindhoven, the Netherlands

Ray J. Dolan (s.fischer@ucl.ac.uk)

Max Planck UCL Centre for Comp. Psych. and Ageing Res., University College London, 10-12 Russell Square
London, WC1B 5EH, United Kingdom

Hartwig R. Siebner (h.siebner@drcmr.dk)

Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre, Kettegaard Allé 30
2650 Hvidovre, Denmark

Oliver J. Hulme (oliverh@drcmr.dk)

Danish Research Centre for Magnetic Resonance, Copenhagen University Hospital Hvidovre, Kettegaard Allé 30
2650 Hvidovre, Denmark

Abstract

Ergodicity describes an equivalence between the expectation value and the time average of observables. Applied to human behaviour, ergodic theory reveals how individuals should tolerate risk in different environments. To optimise wealth over time, agents should adapt their utility function according to the dynamical setting they face. Linear utility is optimal for additive dynamics, whereas logarithmic utility is optimal for multiplicative dynamics. Whether humans approximate time optimal behavior across different dynamics is unknown. Here we compare the effects of additive versus multiplicative gamble dynamics on risky choice. We show that utility functions are modulated by gamble dynamics in ways not explained by prevailing economic theory. Instead, as predicted by time optimality, risk aversion increases under multiplicative dynamics, distributing close to the values that maximise the time average growth of wealth. We suggest that our findings motivate a need for explicitly grounding theories of decision-making on ergodic considerations.

Keywords: Ergodicity; Expected Utility Theory; Prospect Theory; experimental economics; behavioral economics

Introduction

A physical observable is ergodic if the average over its possible states (an expectation value), is the same as its average over time (a time average). The relevance of ergodicity to human behavior is that it provides important constraints for thinking about how agents should compute averages when making decisions (Peters & Adamou, 2018). In the behavioral sciences, decision making is studied predominantly using experiments where choice outcomes exert additive effects on wealth. In these examples, changes in wealth are ergodic, and a linear utility function is

optimal for maximising the growth of wealth over time (Peters & Adamou, 2018). In other words, for this utility function, when changes in expected utility are maximized per unit time, this maximizes the time average growth rate of wealth. In contrast, settings with multiplicative dynamics have non-ergodic wealth changes, which means that the expectation value of changes in wealth no longer reflect time-average growth. In such multiplicative settings a logarithmic utility function is time optimal, since maximizing changes in expected utility per unit time then maximises the time average growth rate of wealth (Peters & Adamou, 2018). Prevailing formulations of utility theory, including Expected Utility theory (von Neumann & Morgenstern, 1947) and Prospect theory (Kahneman & Tversky, 1979), treat all possible dynamics as the same and imply that utility functions are indifferent to the dynamics. From an ergodic perspective, utility functions do not represent idiosyncratic preferences but rather arise as the ergodicity mappings that agents should apply as they attempt to grow their wealth over time. In other words, utility functions appear as the transformations required to obtain ergodic observables, which when maximised, maximise time average growth of wealth (Peters & Adamou, 2018). Since standard economic theories assume stable but idiosyncratic utility functions, whereas time optimality prescribes specific utility functions for specific dynamics, the two classes of theory make different predictions. Here we experimentally manipulated the ergodic properties of a simple gambling environment. We found convergent evidence that dynamics impose a strong and consistent effect on utility functions, and the pattern of these effects are better approximated by a time optimal model compared to other standard utility models.

Methods

Experimental Setup

20 subjects (2 excluded) engaged in a gambling paradigm with either additive or multiplicative wealth dynamics. Each day they were endowed with an initial wealth of 1000DKK / ~$155 (Fig. 1a), after which they took part in a passive session during which they learned the deterministic effect of nine different fractal stimuli on their endowed wealth. On the additive day (Day+) the fractals caused additive changes in wealth (range +428 to -428DKK) whereas on the multiplicative day (Day×) the fractals caused multiplicative changes to their endowed wealth (~halving to ~doubling). In a subsequent active session, subjects chose between gambles composed of the same fractals.
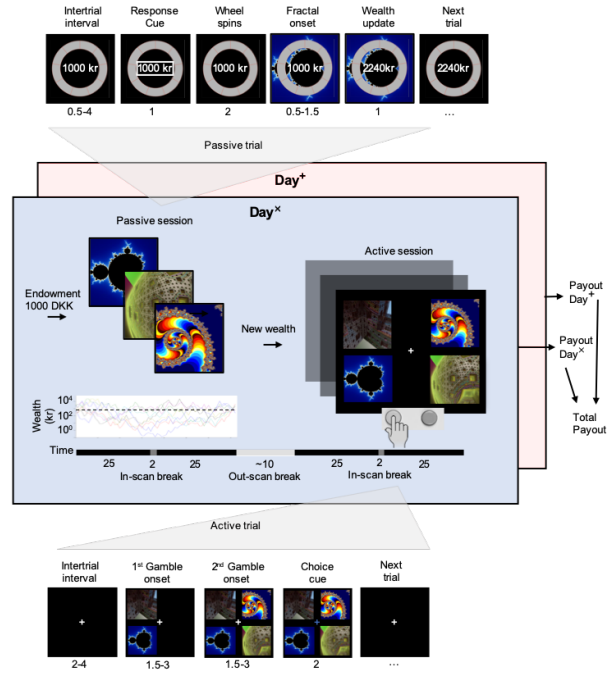

Figure 1: Experimental design. Protocol for both days, differing in the dynamics of wealth changes. Numbers indicate durations in minutes. Top: A single trial from a passive session. Durations are in seconds, ranges depict a uniformly distributed temporal jitter. Bottom: A single trial from an active session.

Dynamics

The dynamics in wealth for the additive and multiplicative day can be expressed as:

$$x(t + \delta t) = x(t) + s(t), \qquad (eq.\,1)$$
$$x(t + \delta t) = x(t) \times s(t), \qquad (eq.\,2)$$

where $s(t)$ is a random variable comprising multiplicative wealth changes on Day×, and additive wealth changes on Day+. The finite time average growth of wealth on Day+ can be calculated as:

$$\bar{g}^+_{\Delta t} = \frac{\Delta x}{\Delta t}, \qquad (eq.\,3)$$

where $\Delta x = x(t_0 + T\delta t) - x(t_0)$, and $\Delta t = T\delta t$. On Day× this is calculated as:

$$\bar{g}^\times_{\Delta t} = \frac{\Delta \ln x}{\Delta t}. \qquad (eq.\,4)$$

The time average additive growth rate for a gamble presented on the left or right side is:

$$\bar{g}^{+(side)} = \left\langle \frac{s^{(side)}}{\delta t} \right\rangle, \qquad (eq.\,5)$$

The time average multiplicative growth rate is:

$$\bar{g}^{\times(side)} = \left\langle \frac{\ln s^{(side)}}{\delta t} \right\rangle, \qquad (eq.\,6)$$

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Utility Models

We modelled choice behavior with different utility models as defined below.

Prospect Theory

$$\delta u = \begin{cases} (\delta x)^{\alpha} & if\ \delta x > 0 \\ -\lambda |(\delta x)|^{\alpha} & if\ \delta x \leq 0 \end{cases} , \qquad (eq.7)$$

where α is a risk preference parameter which lies on the interval $(0,1)$, and λ is a loss aversion parameter which lies on the interval $(0, \infty)$.

Isoelastic Utility

$$\delta u = \delta x \cdot x^{\eta} , \qquad (eq.8)$$

where η reflects risk aversion with risk aversion increasing $\eta > 0$, and risk seeking increasing for $\eta < 0$.

Time Optimal Utility

$$\delta u = \begin{cases} \delta x & if\ additive\ dynamics \\ \delta \ln(x) & if\ multiplicative\ dynamics \end{cases} . \qquad (eq.9)$$

Expected utility For each gamble the expected utility is calculated for each utility model:

$$\langle \delta u^{side} \rangle = 0.5 \cdot \delta u_1{}^{side} + 0.5 \cdot \delta u_2{}^{side} \qquad (eq.10)$$

Stochastic choice function (identical for all models):

$$\theta(\langle \delta u \rangle^{\Delta}) = \frac{1}{1 + e^{-\beta \langle \delta u \rangle^{\Delta}}} , \qquad (eq.11)$$

where $\langle \delta u \rangle^{\Delta}$ is the difference in utility between the gambles, β is a sensitivity parameter, and θ evaluates to the probability of choosing the left-hand gamble.

Data Analysis

Bayesian T-tests were performed with JASP (v0.9.0.1), Bayesian hierarchical modelling with JAGS (v4.03) with 10 chains of 5×10^3 samples, thinning factor 10, burn-in 500.

Results

Time Optimality Utility Model

The isoelastic utility model's parameter space contains values that are time optimal solutions for both additive (η=0) and multiplicative dynamics (η=1, log utility). Fitting a hierarchical Bayesian model, we find strong evidence that risk aversion increases from additive to multiplicative dynamics (paired-t, $BF_{10} = 2.9 \times 10^7$, M_{Δ} =1.001, SD = 0.345), which is indistinguishable from the predicted size of change in η under time optimality. The frequency histograms of η marginalised over all subjects (Fig. 2a) show that the maximum a priori (MAP_{η}) value approximates the time optimal predictions: under additive dynamics, the distribution has a MAP_{η} = .1506 (time optimal prediction: η = 0); under multiplicative dynamics, the distribution has a

MAP_{η}=1.1534, (time optimal prediction: η = 1). The joint distribution over a risk aversion space (Fig. 2b) shows that the MAP estimate is likewise close to the optimal point indicated by the intersection of the prediction lines. This indicates a qualitative agreement between the distribution of risk aversions, and the normative predictions of the time optimality model.
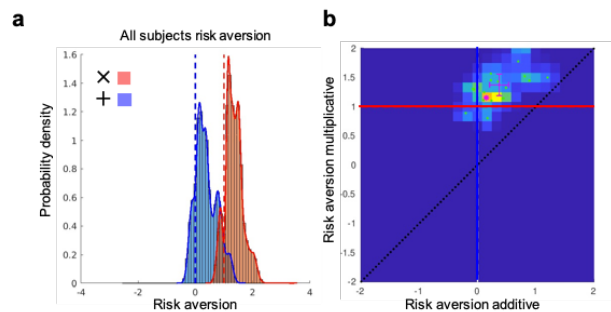

Figure 2. Hierarchical Bayesian modelling for estimating dynamic-specific risk preference. **a,** Frequency distribution of risk aversion values collapsed over subjects for additive (blue) and multiplicative (red) dynamics. Dotted lines indicate time optimal values of risk aversion. **b,** The joint distribution of dynamic-specific risk preferences. MAP values are plotted for the group (pink dot), and for each subject (cyan dots), and are superimposed over the group-level frequency distribution. Error-bars indicate the central Bayesian central credibility interval $BCI_{95\%}$ for the subject-specific MAP values. Time optimal parameter values are at the intersection of the red and blue line. The diagonal line indicates the subspace predicted under a dynamic-invariant model of risk preferences.

Time Optimality against other Utility Models

In a next step, we compared the predictive adequacy of three models in a hierarchical latent mixture model: an Isoelastic model (eq. 8) where η is free to vary between dynamics, a Prospect theory model (eq. 7), and the Time optimal model (eq. 9) with fixed population means of η=0 for additive and η=1 for multiplicative dynamics but where the variance around this mean is a free parameter. Sampling this model results in posterior probabilities for each model, estimated for each subject (Fig. 3a). Most subjects had most of their probability mass located over the time optimal model (Fig. 3b). The time optimal model had an exceedance probability of 0.976 (Fig. 3c) which corresponds to very strong evidence for being the most frequent ($BF_{Time-PT} = 76.9$, $BF_{Time-Iso}$ 80.6).
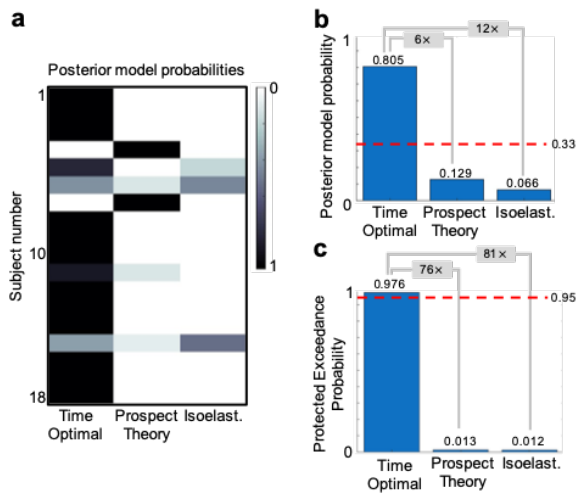
Figure 3. Bayesian hierarchical latent mixture model. **a,** Posterior model probabilities for each model. **b**, Posterior model probabilities summed over subjects, with the red bar indicating prior probabilities assuming equal prior probability for the three utility models. **c,** Protected exceedance probabilities for each utility model being the most frequent.

Discussion

By manipulating the dynamical properties of simple gambles, we show that ergodicity breaking can exert strong and systematic effects on human behavior. Switching from additive to multiplicative dynamics reliably increased risk aversion, which tracked close to the levels that maximise the time average growth of wealth. We show that these effects cannot be adequately explained by the prevailing models of utility in economics and psychology, and are well approximated by a null model of time optimality.

The time optimal model assumes that agents have a stable preference for their wealth to grow faster. Consequently, to maximise the time average growth rate of wealth, agents should adapt their utility functions according to the wealth dynamics, such that changes in utility are rendered ergodic (Peters & Adamou, 2018). From this, a number of predictions can be derived. First, different dynamics should evoke the observation of a different utility function. This was observed in all subjects. Second, in shifting from additive to multiplicative dynamics, agents should become more risk averse. This too was observed in all subjects. Third, the predicted increase in risk aversion should be a step change of +1. The mean step change across the group was +1.001 (BCI$_{95\%}$ [0.829,1.172]). Finally, most participants had ~linear utility under additive dynamics and ~logarithmic utility under multiplicative dynamics.

Bayesian model comparison revealed strong evidence for the time optimal model compared to both Prospect theory and Isoelastic utility models. The latter two provide no explanation or prediction for how risk preferences should change when gamble dynamics change, and even formally preclude the possibility of maximising the time average growth rate when gamble dynamics do change.

Our observations are relevant to a widespread assumption in economic theory that preferences are stable over time (Stigler & Becker, 1977). If utility is to predict behaviour in future settings, then it must be stable, otherwise if behavior changes, it is not known if this is due a change of setting or preference, or both (Andersen, Harrison, Lau, & Rutström, 2008). Our findings cast the dynamical dependence of utility functions not as preference instability per se, but simply as a manifestation of a stable preference for growing wealth over time. Together, this motivates a need to explicitly condition theories of decision making, and their applications, on ergodic considerations.

References

Andersen, S., Harrison, G. W., Lau, M. I., & Rutström, E. E. (2008). Lost in State Space: Are Preferences Stable?*. *International Economic Review*, *49*(3), 1091–1112.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, *47*, 263–291.

Peters, O., & Adamou, A. (2018). The time interpretation of expected utility theory. *ArXiv:1801.03680 [q-Fin]*.

Peters, O., & Gell-Mann, M. (2016). Evaluating gambles using dynamics. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *26*(2), 023103.

Stigler, G., & Becker, G. (1977). De Gustibus Non Est Disputandum. *American Economic Review*, *67*(2), 76–90.

von Neumann, J., & Morgenstern, O. (1947). *The Theory of Games and Economic Behavior*. Princeton University Press.